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Omnidirectional Reflection Band Gap in Single Composite Layer of Negative Index Material

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Abstract

The enlarged omni-directional reflection band gap (ODRBG) of a semi-infinite single layer of negative index material (NIM) has been investigated. We know that the effective refractive index of the NIM depends on electric permittivity and magnetic permeability. The zero refractive index of the NIM is a phenomenal behavior for the plasma frequency of the material. We have calculated the optical properties of the considered material with varying the magnetic plasma frequency. The reflectance of the single layer for TE and TM modes is obtained 100% omni-directional reflection band gap (ODRBG) when the refractive index of the NIM is zero. The enlarged ODRBG is found when the magnetic plasma frequency increases.

Keywords: Semi-infinite single layer material, negative index material, zero-refractive index, omni-directional reflection band.

1. Introduction

The optical properties of the periodic structure are studied at nano-scale for various optical applications, known as photonic crystal (PC). The periodic nano patterns of the photonic crystals have the photonic band gap, which is analogous to the electronic band gap in semiconductors. Such photonic crystal may be existed negative refractive index that is called metamaterials or negative index material (NIM). In 1968, first time Veselago[1] proposed a new material that has negative permittivity ($-\epsilon$) and the negative permeability ($-\mu$) simultaneously at certain frequency, which has a peculiar and unusual medium due to negative refractive index and also called left-handed material.

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Experimentally, it has been observed that the value of negative permittivity ($-\epsilon$) and negative permeability ($-\mu$) of the NIM are dependent upon the incident frequency and the plasma frequency of the material [2, 3]. Other kind of materials are also reported which have either negative permittivity ($-\epsilon$) with permeability (μ) constant or negative permeability ($-\mu$) with permittivity (ϵ) constant, called the single-negative (SNG) material. The possible materials of SNG are: (i) $\epsilon < 0$ (ENG) and $\mu > 0$ (ii) $\epsilon > 0$ and $\mu < 0$ (MNG). The single negative materials have a unique properties compared with the double negative materials of the NIM. We know that the photonic crystals (PCs) are the artificial materials with periodic modulation of the dielectric constants. The photonic band-gap structures are dependent upon the symmetry, dielectric constant of material, and the scale length of crystal lattice of the PCs. The band structure can also be changed with the ϵ and μ of the materials, because the refractive index is related with ϵ and μ , $n^2 = \epsilon\mu$ [4].

The transmission properties of photonic crystals consisting of μ -negative material (MNG) and positive-index material (PIM) have shown the transmission band inside a single-negative gap. The width of the transmission band is only dependent on the thickness of the MNG layers. The tunneling mode is localized strongly inside the PIM layers with the increasing in the thickness of the μ -negative layers. The transmittance can be increased by decreasing the thickness of the MNG layers [5]. The heterostructure photonic structure, composed with two different PCs, can be used as a tunable zero-phase-shift for omnidirectional filter. By simply adjusting the thickness of the defect layer of air, the adjustability of the tunneling mode can be achieved. The omnidirectional tunneling modes in the heterostructure photonic structure consisting of layered single-negative materials and one layer of defect has been discussed by Tong [6]. The two kinds of photonic crystals with single-negative materials have observed for zero-effective phase gap as well as angular gap. The zero-effective phase gap is invariant with the incident angle and lattice scaling for different polarizations. The gap of the structure can be adjusted by varying the ratio of thicknesses of the two single negative materials. The angular gap is highly dependent on the incident angle and polarization, but it is insensitive to the ratio between thickness of the two single negative materials and lattice scale length [7]. The propagation of light waves in one-dimensional photonic crystals (1DPCs) composed of alternating layers of two kinds of single-negative materials have shown that the phase velocity is found negative when the

frequency of the light wave is smaller than the certain critical frequency ω_{cr} [8].

The thickness-dependent photonic band gap for a one-dimensional photonic crystal consisting of two different single-negative (SNG) materials studied theoretically by Yeh and Wu [9]. The two SNG materials are considered with the single-negative permittivity ($\epsilon < 0$, $\mu > 0$) and the single-negative permeability ($\epsilon > 0$, $\mu < 0$). The considered structures have calculated the size of the band gap and the positions of the band edges which are strongly dependent on the thickness ratio of the two constituent SNG layers. By using the composite right/left-hand transmission line model, the shifting behavior of band gap has qualitatively explained due to the variation of the thickness.

The transverse-electric (TE) wave propagation through lossless tri-layer stacks containing single-negative (SNG) materials i.e. negatively permittivity (ϵ) or negatively permeability (μ), has analyzed by considering the following combinations: ENG/MNG/ENG, ENG/DPS/MNG, DPS/ENG/DPS, and ENG/DPS/ENG, where ENG refers to epsilon-negative, MNG to mu-negative, and DPS to double-positive media [10]. The reflection phase difference between TE and TM waves in the one-dimensional photonic crystals composed of single-negative (SNG) materials have been observed the two omni-directional gaps where the reflection phase difference is changed smoothly and increased with the increasing of the incident angles [11].

Castaldi et al. [12] have shown that resonant tunneling of electromagnetic fields can be occurred through a three-layer structure composed of the single-negative(i.e., either negative permittivity or negative permeability) slab paired with a bilayer made of double-positive(i.e., positive permittivity and permeability) media. The study results is demonstrated that the counter intuitive tunneling phenomenon is also the possibility of synthesizing double-positive slabs which is effectively exhibited single-negative-like wave-impedance properties within a moderately wide frequency range. The band structure and band gaps of the one-dimensional Fibonacci quasi crystals composed of epsilon-negative materials and mu-negative materials are also studied and shown an omni-directional band gap (OBG). In contrast to the Bragg gaps, such the OBG is found insensitive to the incident angles and the polarizations of light. The width and location of the OBG is ceased with increasing Fibonacci order, but varied with the thickness ratio of both components. The OBG is closed when the thickness ratio is equal to the golden ratio [13].

The enlarged band gap can be obtained when the high ϵ and the large thickness of MNG as well as the high μ and the ENG with small thickness of ENG are considered. The

study of the transmittance shown that the tunneling property of electromagnetic wave is existed due to presence of zero- ϵ and zero- μ . By choosing proper thicknesses of the structure and angles of incidence, the large omni-directional band gap can be achieved which can use as a filter [14, 15].

The SNG has unique properties compared to double negative materials but the zero-refractive index play important role for omnidirectional reflection band. For negative refractive index, the plasma frequencies always consider larger value than the incident frequency of wave. We have calculated optical density and reflectance characteristics of single composite layer of negative index material by changing magnetic plasma frequency. The magnetic plasma frequency of NIM is affected the zero-refractive index because large plasma frequency is changed large value of the zero refractive index. The large value of the zero refractive index of NIM is responsible for enlarged omnidirectional reflection band gap in single composite layer of negative index material.

2. Methodology and Physical Model

According to Maxwell's equations of the electromagnetic theory, the square of the refractive index of the material is equal to the product of electric permittivity (ϵ) and magnetic permeability (μ) of the material. So refractive index is written as,

$$n^2 = \epsilon\mu \quad (1)$$

Where ϵ and μ are the electric permittivity and magnetic permeability respectively. For calculation of the optical density and reflection of the single composite layer of NIM, we have taken $\epsilon = 1 - \frac{(\omega_{ep}^2 - \omega_{e0}^2)}{(\omega^2 - \omega_{e0}^2 + i\gamma\omega)}$ and $\mu = 1 - \frac{(\omega_{mp}^2 - \omega_{m0}^2)}{(\omega^2 - \omega_{m0}^2 + i\gamma\omega)}$ where ω_{ep} =electric plasma frequency, ω_{mp} =magnetic plasma frequency, ω_{e0} =natural electric frequency, ω_{m0} =natural magnetic frequency, γ =attenuation and ω =incident frequency of the wave [15].

To calculate optical density, transmittance and reflectance of the single composite layer with negative index material, we consider a single composite layer material of NIM with thickness d which is sandwiched between two semi-infinite dielectric media like air and substrate. The refractive index distribution along x-axis is given as below

$$n(x) = \begin{cases} n_0 & x < 0 & \text{(air)} \\ n_{NIM} & 0 < x < d & \text{(NIM)} \\ n_s & d < x & \text{(substrate)} \end{cases} \quad (2)$$

The electromagnetic wave is incident in the x-y plane, so the plane wave equation for the electric field is defined by $E(x,t) = E(x)\exp[i(\omega t - \beta x)]$, where β is the y-component of the propagation wave vector. By considering the plane wave incident from the left side, the electric field for the three regions is given as;

$$E(x) = \begin{cases} Ae^{-ik_{\text{air}}x} + Be^{ik_{\text{air}}x} & x < 0 & \text{(air)} \\ Ce^{-ik_{\text{NIM}}x} + De^{ik_{\text{NIM}}x} & 0 < x < d & \text{(NIM)} \\ Fe^{ik_{\text{sub}}(x-d)} & d < x & \text{(substrate)} \end{cases} \quad (3)$$

where A,B,C,D and F are constants and k_{air} , k_{NIM} , and k_{sub} are the x-components of wave vector in n_0 , n_{NIM} , n_{sub} respectively. The wave vector k_{ix} is defined as $k_{ix} = \left[\left(\frac{n_i \omega}{c} \right)^2 - \beta^2 \right]^{1/2} = \frac{\omega n_i}{c} \cos \theta_i$, where $i=0, \text{NIM, sub}$; and θ_i is the ray angle measured from x-axis. The magnetic field of the corresponding electric field is defined by;

$$H = \frac{i}{\omega c} \nabla \times E \quad (4)$$

By applying boundary conditions on the tangential component of electric field (TE-mode) and the tangential component of magnetic field (TM-mode), we are considered that there is no coupling between these components in the whole medium. The boundary condition at $x=0$ and $x=d$ is applied in equations (3-4) and we can obtain the transmission coefficient [16, 17];

$$t = F/A = \frac{4k_{0x}k_{\text{sup}}e^{-ik_{\text{sup}}d}}{(k_{0x}+k_{\text{sup}})(k_{\text{sup}}+k_{\text{sx}}) \left[1 + \frac{(k_{0x}-k_{\text{sup}})(k_{\text{sup}}-k_{\text{sx}})e^{-2ik_{\text{sup}}d}}{(k_{0x}+k_{\text{sup}})(k_{\text{sup}}+k_{\text{sx}})} \right]} \text{ or } t = \frac{t_{12}t_{23}e^{-ik_{\text{sup}}d}}{[1+r_{12}r_{23}e^{-2ik_{\text{sup}}d}]} \quad (5)$$

Similarly, the reflection coefficient is given as;

$$r = B/A = \frac{[(k_{\text{sup}}-k_{0x})(k_{\text{sup}}+k_{\text{sx}}) + (k_{0x}+k_{\text{sup}})(k_{\text{sx}}-k_{\text{sup}})e^{-2ik_{\text{sup}}d}]}{[(k_{0x}-k_{\text{sup}})(k_{\text{sx}}-k_{\text{sup}})e^{-2ik_{\text{sup}}d} - (k_{0x}+k_{\text{sup}})(k_{\text{sup}}+k_{\text{sx}})]} \text{ or } r = \frac{r_{12}+r_{23}e^{-2ik_{\text{sup}}d}}{1+r_{12}r_{23}e^{-2ik_{\text{sup}}d}} \quad (6)$$

We have used equation (1) to calculate optical density of the single composite NIM and equation (6) is used to calculate the reflectance of the considered structure.

3. Results and Discussion

The single composite layer of the negative index materials (NIM) has the electric permittivity (ϵ) and the magnetic permeability (μ) and these parameters are related with the plasma frequency, incident frequency and attenuation which is given as;

$$\epsilon = 1 - \frac{(\omega_{\text{ep}}^2 - \omega_{\text{e0}}^2)}{(\omega^2 - \omega_{\text{e0}}^2 + i\gamma\omega)} \text{ and } \mu = 1 - \frac{(\omega_{\text{mp}}^2 - \omega_{\text{m0}}^2)}{(\omega^2 - \omega_{\text{m0}}^2 + i\gamma\omega)} \quad (7)$$

where $\omega_{\text{ep}}=8.0425 \times 10^{10}$ rad/sec., $\omega_{\text{mp}}=6.8801 \times 10^{10}$ rad/sec., $\omega_{\text{e0}}=6.4717 \times 10^{10}$ rad/sec., $\omega_{\text{m0}}=6.3146 \times 10^{10}$ rad./sec. and $\gamma=6283.2 \times 10^{10}$ rad/sec. The incidence frequency range is considered from 6×10^{10} rad/sec. to 18×10^{10} rad/sec. and refractive indices of air and substrate

are unity i.e. $n=1$ [15]. Now putting the values of the electric permittivity (ϵ) and magnetic permeability (μ) in equation (1), we obtained the optical density of the NIM as shown in Fig. 1. The blue line shows the refractive index of the NIM and green line shows the extinction coefficient of the NIM. The optical density (n) has obtained three parts of the frequency range. The negative value of refractive index (n) is found when below the $\omega=7.1 \times 10^{10}$ rad./sec. and positive refractive index (n) is found when above $\omega=8.0 \times 10^{10}$ rad./sec. But the zero-refractive index is observed when the frequency range lies between $\omega=7.1 \times 10^{10}$ rad./sec. to 8.0×10^{10} rad./sec.

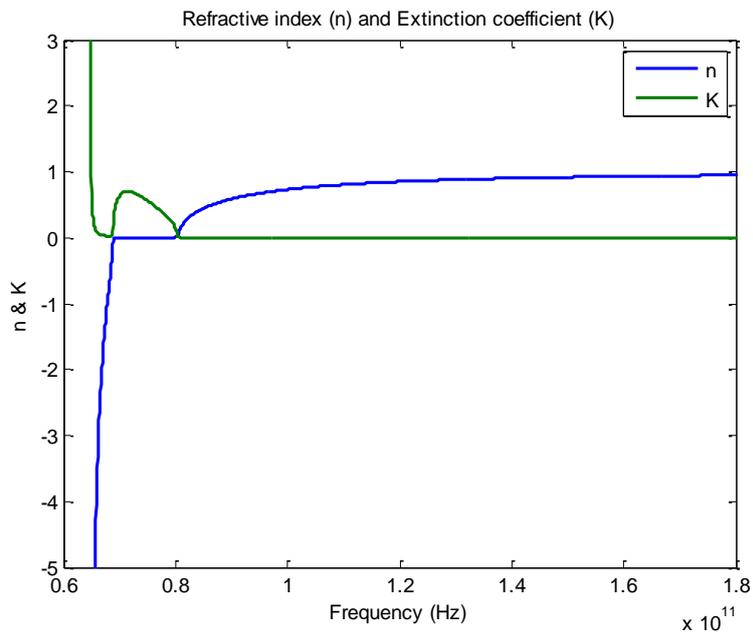


Figure 1: Optical density (n) and extinction coefficient (K) of the NIM

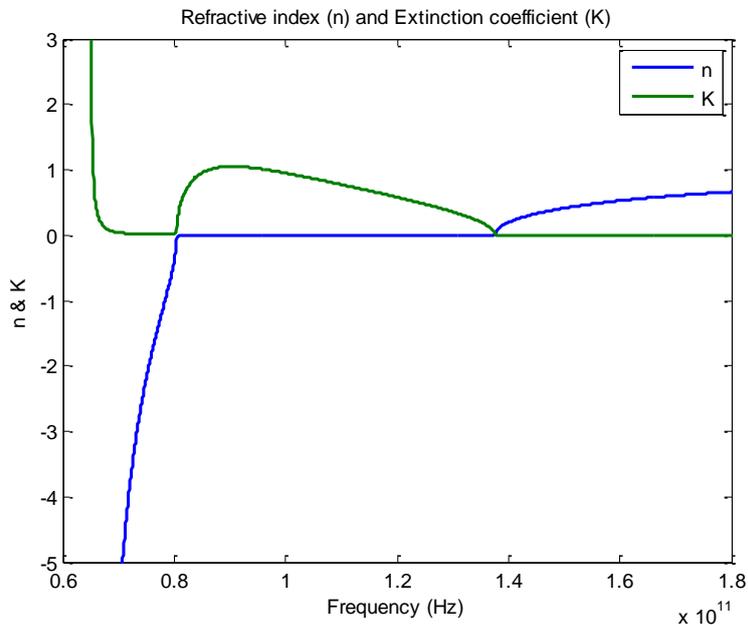
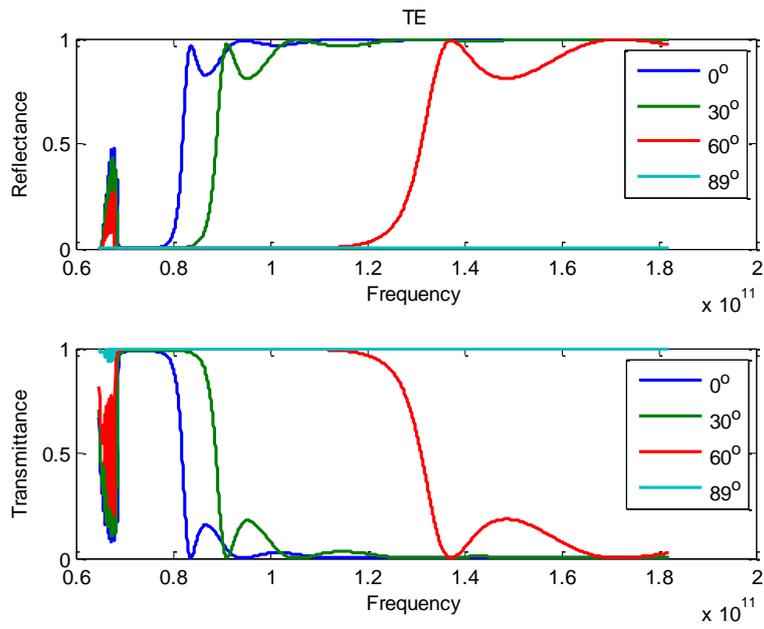


Figure 2: Optical density (n) and extinction coefficient (K) of the NIM when plasma frequency is double.

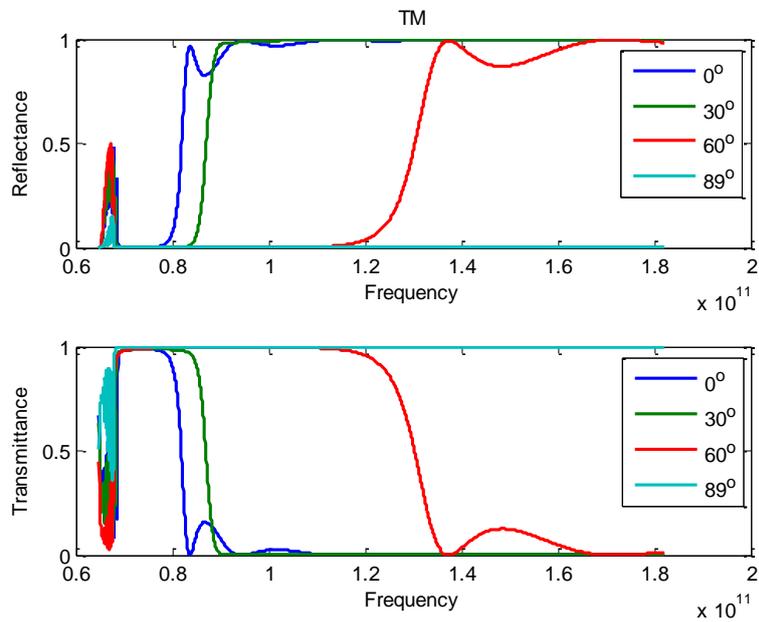
The extinction coefficient of the NIM is very high for negative-n and nearly zero value for positive-n. For the zero-refractive index has shown for some amount of the extinction coefficient which shows absorption in this region. We have studied that the magnetic plasma frequency of the NIM affects the zero-refractive index compare to electric plasma frequency. So we increase the magnetic plasma frequency double times of the considered ω_{mp} i.e. 13.7602×10^{10} rad./sec. which is double of the previous calculation, then we obtain a large zero refractive index of frequency range from 8.1×10^{10} rad./sec. to 14×10^{10} rad./sec. as shown in the Fig. 2.

In literature survey we have been studied that the zero refractive index of the NIM has independent upon the polarizations and have 100% reflectance for the TE and TM modes. So we have calculated the reflectance and the transmittance of the NIM for TE and TM modes versus frequency with varying incidence angles using equation (6) which is shown Fig. 3(a) for TE mode and Fig. 3(b) for TM mode. The 100% reflectance of the single composite layer of the NIM for both modes is obtained at the same frequency range where the refractive index obtained zero i.e. the zero-refractive index is found from $\omega = 7.1 \times 10^{10}$ rad./sec. to 8.0×10^{10} rad./sec. The Figs. 3 show that the reflectance of the single layer of the NIM increases when the angles of incidence are increased. The reflectance of the considered structure has found 0% reflectance and other side the transmittance of the same structure is found 100% for the highest angle of incidence i.e. 89° .

The Fig. 4 shows the reflectance and the transmittance of the single composite layer of NIM when $\omega_{mp} = 13.7602 \times 10^{10}$ rad./sec. which is just double of the previous calculation. The reflectance band gap is obtained where the refractive index has obtained zero at the same range of the frequency 7.1×10^{10} rad./sec. to 14×10^{10} rad./sec. So the reflectance band gap is about $\Delta\omega = 6 \times 10^{10}$ rad./sec. which is six times larger than first calculation when the magnetic plasma frequency is double. These calculations reveal that the enlarged reflectance band gap of the single NIM is obtained when the magnetic plasma frequency of NIM increases. The reflectance and the transmittance of the single layer of the NIM with plasma frequency (ω_{mp}) 13.7602×10^{10} rad/sec. are changed with varying angles of incidence from 0° to 89° . The band gap of the material is obtained approximately $\Delta\omega = 6 \times 10^{10}$ rad/sec. Such structure may be used as the filter due to large omnidirectional reflection band gap in both modes.



(a)



(b)

Figure 3: Reflectance and transmittance of single composite layer of NIM versus frequency when $\omega_{mp}=6.8801 \times 10^{10}$ rad./sec. for (a) TE-mode and (b) TM-mode.

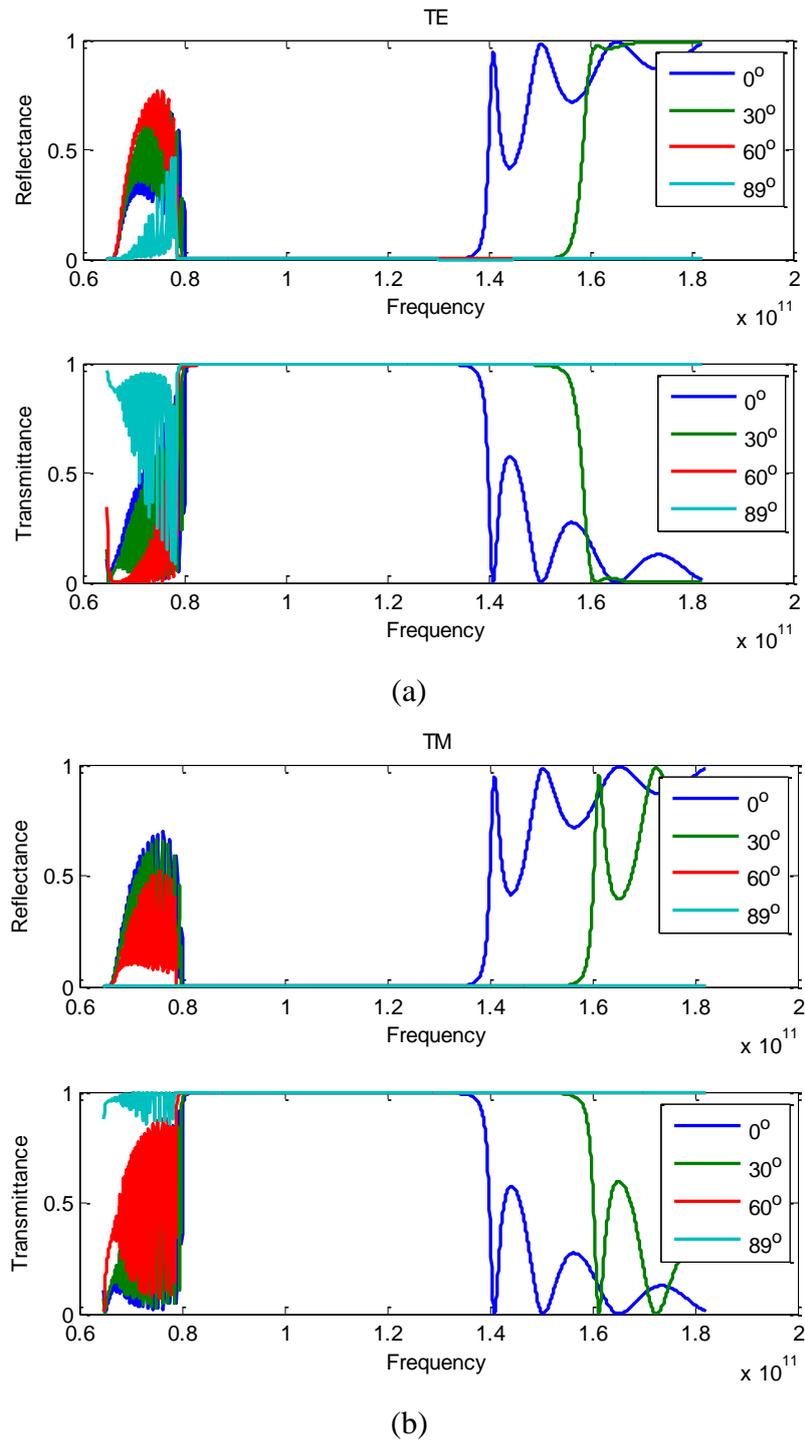


Figure 4: Reflectance and transmittance of single composite layer of NIM versus frequency when $\omega_{mp}=13.7602 \times 10^{10}$ rad./sec. for (a) TE-mode and (b) TM-mode.

4. Conclusions

We have studied the optical characteristics single composite layer of NIM with varying the magnetic plasma frequency as well as the angles of incidence. The increased

magnetic plasma frequency of NIM is enlarged six times of the zero refractive index when magnetic plasma frequency is increased two times only. The calculation shows that the zero refractive index of the NIM is independent of the polarizations and dependent upon angles of incidence. The reflectance and the transmittance of the NIM is enlarged due to enlarged of the zero refractive index. The reflection band gap of the NIM with large magnetic plasma frequency is also independent of the angles of incidence. Such enlarged reflection band gap of the single layer of the NIM with large magnetic plasma frequency may be used as the filter due to obtain enlarged omnidirectional reflection band gap in both modes.

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