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One-Dimensional Photonic Crystals and Computational Techniques: A Pedagogical Review

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Abstract

Photonic crystals are optical media with periodic variation of the dielectric function. The sequence of layering and choice of material of various layers determine electromagnetic wave propagation in these structures. A pedagogical review of the physics and behaviour of one-dimensional photonic crystals, in which layers of dielectric media are arranged with periodicity only in one direction, is presented here. Theoretical underpinnings, as well as widely used numerical and computational techniques viz., Transfer Matrix Method (TMM), Scattering Matrix Method (SMM) and Finite Difference Time Domain Method (FDTD) have been reviewed here. A detailed derivation of the transfer matrix method has been given, along with the summaries of SMM, and finite-difference time-domain approaches. Binary and ternary photonic crystal configurations are briefly discussed, and key experimental milestones are reviewed. The article is intended as a self-contained introduction for students and newcomers to the field.

Keywords: Photonic Crystals, Transfer Matrix Method, Photonic Bandgap, Bragg Reflectors, Multilayer Structures

1 Introduction

The idea that electromagnetic waves in a periodic medium encounter forbidden frequency ranges dates back to 1887. Lord Rayleigh, in a paper [1] titled "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," showed that a one-dimensional medium with even an infinitesimally small periodic variation in dielectric properties produces a forbidden frequency band in which waves cannot propagate and are totally reflected. In a follow-up paper in 1888 [2], he noted that this can effectively behave as a total reflection mirror. The study of electromagnetic wave propagation in periodic media has a century long history. A brief account of it is given here [3] and summarized in table 1

The next step after the Rayleigh's work in 1887 came from X-ray crystallography, in 1914 by C.G. Darwin who worked out a theory of X-ray diffraction in crystals that included finite stop bands [4]. The question of what a photonic bandgap does to light emission has also been addressed by V.P. Bykov, a Soviet physicist [5]. In 1972 and 1975, he studied spontaneous emission from atoms and molecules embedded inside a one-dimensional periodic dielectric structure. In 1979, Kazuo Ohtaka [6] in Japan calculated the photonic band structure for electromagnetic waves in a three-dimensional lattice of dielectric spheres. The modern era of photonic crystals began in 1987 with publication of two independent papers [7,8]. One of these papers was authored by Eli Yablonovitch, then at Bell Communications Research. He was trying to improve semiconductor lasers by reducing unwanted spontaneous emission losses. He proposed that a three-dimensional periodic dielectric structure could create a frequency range in which no electromagnetic modes exist, and thus may suppress spontaneous emission altogether. The second paper was published by Sajeev John, then an assistant professor at Princeton University. He independently proposed [8] that photons could undergo Anderson localization in disordered dielectric superlattices.

Following period witnessed theoretical and experimental efforts to actually demonstrate a complete three-dimensional photonic bandgap. For example, Yablonovitch and Gmitter reported a candidate fcc structure in 1989 [16]. In 1990, Ho, Chan and Soukoulis [9] demonstrated theoretically that the diamond structure does have a complete gap. Again, Yablonovitch, Gmitter and Leung [10] then fabricated such a structure — now called Yablonovite and confirmed the complete bandgap experimentally in 1991. Krauss and De La Rue [12] demonstrated the first two-dimensional photonic crystal operating at near-infrared wavelengths in 1996, using semiconductor fabrication. In 1998, Fink *et al.* [13] showed the conditions of omnidirectional reflection even in a one-dimensional photonic crystal. Lin *et al.* demonstrated a three-dimensional photonic crystal which was operating at infrared wavelengths [16].

It is worth mentioning that one-dimensional photonic crystals had been in practical

Table 1: Key milestones in the development of photonic crystal physics.

Year	Contributor(s)	Contribution	Ref.
1887	Lord Rayleigh	Showed that a one-dimensional periodic medium has a forbidden frequency band (stop band), and derived its width in terms of the modulation strength and period.	[1]
1888	Lord Rayleigh	Showed that such a structure acts as a total reflection mirror, even for infinitesimally small periodic variation.	[2]
1914	C. G. Darwin	Developed the dynamical theory of X-ray diffraction in crystals, which included finite stop bands for X-ray wavelengths.	[4]
1972	V. P. Bykov	First to study the effect of a photonic bandgap on spontaneous emission from atoms embedded in a one-dimensional periodic dielectric structure.	[5]
1975	V. P. Bykov	Extended his analysis to media with a band spectrum; speculated about two- and three-dimensional periodic structures.	[5]
1979	K. Ohtaka	Calculated the photonic band structure for electromagnetic waves in a three-dimensional lattice of dielectric spheres, extending the KKR formalism from electronic to photonic band theory.	[6]
1987	E. Yablonovitch	Proposed using three-dimensional periodic dielectric structures to create a complete photonic bandgap and thereby inhibit spontaneous emission.	[7]
1987	S. John	Proposed strong localization of photons in disordered dielectric superlattices via photonic bandgap effects.	[8]
1989	E. Yablonovitch, T. J. Gmitter	First experimental attempt at a three-dimensional photonic bandgap (fcc structure); later found not to possess a complete gap.	[16]
1990	K. M. Ho, C. T. Chan, C. M. Soukoulis	Showed theoretically that the diamond structure has a complete photonic bandgap.	[9]
1991	E. Yablonovitch, T. J. Gmitter, K. M. Leung	Fabricated the first structure with a verified complete three-dimensional photonic bandgap ("Yablonovite").	[10]
1996	T. F. Krauss, R. M. De La Rue, S. Brand	Demonstrated the first two-dimensional photonic crystal operating at near-infrared wavelengths, using semiconductor fabrication.	[12]
1998	Y. Fink, J. N. Winn, S. Fan, J. D. Joannopoulos <i>et al.</i>	Constructed one dimensional photonic crystal as a omnidirectional reflector out of a stack of nine alternating layers of chosen dielectrics.	[13]
1998	S. Y. Lin <i>et al.</i>	Demonstrated a three-dimensional photonic crystal operating at infrared wavelengths.	[16]

use long before any of this terminology existed. By the mid-twentieth century, many technologies like dielectric multilayer mirrors, anti-reflection coatings and interference filters were perfected and in wide use [17]. The post-1987 framework did not, at first, displace these established technologies, but it did supply a unified analytical framework from solid-state physics that connected these previously separate technologies to a common physical picture of bands, gaps, and Bloch modes. The work in 1987 established that the photonic band structure results from the periodic modulation of the refractive index in dielectric materials [7,8]. This emergence of photonic band structure is, in some sense, analogous to the emergence of electronic band structure from the periodic atomic potential in semiconductors.

The simplest example of photonic crystals are the one-dimensional photonic crystals in which one-directional periodicity of dielectric constant is the defining feature. Though their geometry is simple, these structures exhibit interesting properties and have found widespread applications which include high-reflectivity mirrors, optical filters, distributed feedback lasers and biosensors [16]. The one-dimensional case is an ideal system, pedagogically, for understanding fundamental concepts that extend to higher-dimensional photonic crystals while remaining analytically tractable.

Binary photonic crystals are formed when layers of only two dielectric materials are used to form the structure. These structures demonstrate the basic physics of photonic bandgap formation. It was found that if a stack of layers is fabricated in such a way that each layer has an optical thickness of one-quarter wavelength, bandgap width is maximised. This layered structure is known as the quarter-wave stack and has become a standard design in optical coatings [17].

Ternary photonic crystals are formed using three distinct dielectric materials in the periodic unit cell. This gives an additional degree of freedom and enables more control over the properties of the media and allows for multiple band gaps, and enhanced light-matter interaction [18].

This review is organized to provide a self-contained introduction to the field. Section 2 gives the electromagnetic foundation from Maxwell's equations, establishing the wave equation in stratified media. Section 3 presents the transfer matrix method in detail, which has proven to be the primary analytical tool for one-dimensional systems. Section 3 also discusses the scattering matrix method and its application to periodic structures. Numerical approaches, particularly the finite-difference time-domain method is also discussed in this section. Types of one dimensional photonic crystals are briefly described in section 4. Section 5 gives a brief description of fabrication methods and of experimental techniques for characterization. Finally, Section 6 and 7 outline applications and future directions.

2 Electromagnetic Wave Theory in Layered Media

2.1 Maxwell's Equations and Wave Propagation

The behavior of electromagnetic waves in photonic crystals is governed by Maxwell's equations. In SI units, these fundamental equations are:

$$\nabla \cdot \mathbf{D} = \rho_f \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \quad (4)$$

where symbols $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, $\mathbf{B}$, ρ_f , and $\mathbf{J}_f$ carry their usual meaning.

The constitutive relations are:

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \epsilon_r(\mathbf{r}) \mathbf{E}(\mathbf{r}, t) \quad (5)$$

$$\mathbf{B}(\mathbf{r}, t) = \mu_0 \mu_r \mathbf{H}(\mathbf{r}, t) \quad (6)$$

For non-magnetic dielectric materials, which are the primary constituents of photonic crystals, $\mu_r = 1$, so the constitutive relations become:

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \epsilon_r(\mathbf{r}) \mathbf{E}(\mathbf{r}, t) \quad (7)$$

$$\mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{H}(\mathbf{r}, t) \quad (8)$$

where ϵ_0 and μ_0 are the permittivity and permeability of free space, and $\epsilon_r(\mathbf{r})$ is the position-dependent relative permittivity (dielectric function). In photonic crystals, $\epsilon_r(\mathbf{r})$ varies periodically with position.

In source-free regions ($\rho_f = 0$, $\mathbf{J}_f = \mathbf{0}$), taking the curl of Faraday's law and substituting Ampère's law yields the wave equation for the electric field:

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\epsilon_r(\mathbf{r})}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (9)$$

where $c = 1/\sqrt{\epsilon_0 \mu_0}$ is the speed of light in vacuum. For time-harmonic fields $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{-i\omega t}$, this becomes:

$$\nabla \times (\nabla \times \mathbf{E}(\mathbf{r})) = \frac{\omega^2}{c^2} \epsilon_r(\mathbf{r}) \mathbf{E}(\mathbf{r}) \quad (10)$$

An equivalent and more useful form is obtained by working with the magnetic field $\mathbf{H}$:

$$\nabla \times \left(\frac{1}{\epsilon_r(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{r}) \quad (11)$$

This last equation is the master eigenvalue equation for photonic band structure calculations. Its operator $\hat{\Theta} \equiv \nabla \times \frac{1}{\epsilon_r} \nabla \times$ is Hermitian on the space of transverse fields, guaranteeing real eigenvalues ω^2 and an orthogonal set of modes [16].

2.2 One-Dimensional Wave Equation

For one-dimensional photonic crystals, the dielectric function varies only along one direction (conventionally the z -direction): $\epsilon_r(\mathbf{r}) = \epsilon_r(z)$. The structure is invariant under translations in the xy -plane. This symmetry allows us to write solutions as:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(z)e^{i(k_x x + k_y y)} \quad (12)$$

where k_x and k_y are conserved wave vector components.

For a wave incident obliquely, so that $\mathbf{k}_{\parallel} = (k_x, k_y)$ is non-zero, the field can be decomposed into two polarisations which are independent of each other. The plane containing $\mathbf{k}$ and the layer normal is the plane of incidence. One of these is termed the transverse electric (TE) mode, with $\mathbf{E}$ perpendicular to the plane of incidence. The other is termed transverse magnetic (TM), with $\mathbf{H}$ perpendicular to the plane of incidence. At strictly normal incidence ($k_x = k_y = 0$) no plane of incidence is defined and the two polarisations become degenerate; the equations below are written for this case, which is the one used throughout this review.

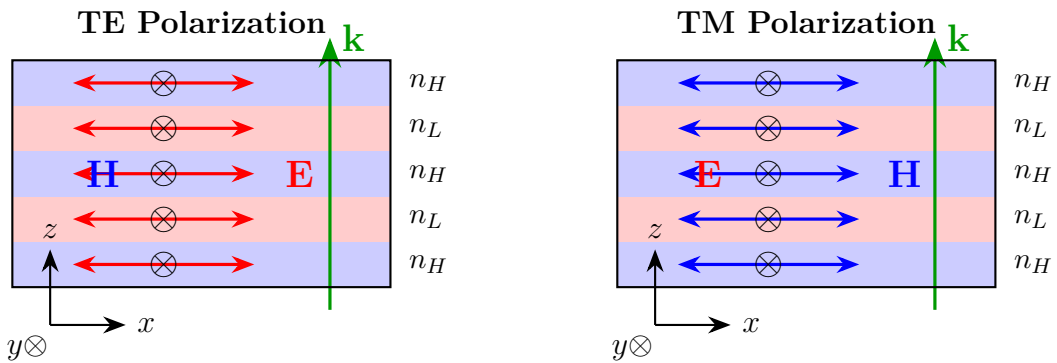


Figure 1: Schematic illustration of TE and TM polarizations for normal incidence on a one-dimensional photonic crystal. The layers alternate between high (n_H) and low (n_L) refractive index materials. For the polarisation labelled TE, the electric field $\mathbf{E}$ oscillates along the x -direction while the magnetic field $\mathbf{H}$ points along y , into and out of the page, shown as $\otimes$. For TM the roles are reversed. The wave vector $\mathbf{k}$ is in the z -direction, perpendicular to the layers; at this strictly normal incidence the two are degenerate, and the labels are retained only to fix the field orientations used in the derivations that follow.

For TE polarization with $\mathbf{E} = E_x(z)\hat{\mathbf{x}}$, the wave equation reduces to:

$$\frac{d^2 E_x}{dz^2} + \epsilon_r(z)k_0^2 E_x = 0 \quad (13)$$

where $k_0 = \omega/c$ is the free-space wave number.

For TM polarization with $\mathbf{H} = H_x(z)\hat{\mathbf{x}}$, the equation is:

$$\frac{d}{dz} \left(\frac{1}{\epsilon_r(z)} \frac{dH_x}{dz} \right) + k_0^2 H_x = 0 \quad (14)$$

These differential equations, subject to appropriate boundary conditions, determine the electromagnetic modes in one-dimensional photonic crystals. The difference between TE and TM polarizations arises from the derivative of the dielectric function in the TM case. This leads to different bandgap characteristics for the two polarizations [19].

2.3 Boundary Conditions

At interfaces between different dielectric materials, the electromagnetic fields must satisfy certain boundary conditions. For non-magnetic materials ($\mu_r = 1$), and in the absence of free surface charge and free surface current, these conditions are:

$$\hat{\mathbf{n}} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0 \quad (15)$$

$$\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0 \quad (16)$$

$$\hat{\mathbf{n}} \cdot (\epsilon_2 \mathbf{E}_2 - \epsilon_1 \mathbf{E}_1) = 0 \quad (17)$$

$$\hat{\mathbf{n}} \cdot (\mathbf{H}_2 - \mathbf{H}_1) = 0 \quad (18)$$

Here, $\hat{\mathbf{n}}$ represents unit vector normal to the interface, and subscripts 1 and 2 denote the two sides of the interface.

These boundary conditions set the premises for constructing solutions to the wave equation in structures involving multilayers. This requires the understanding of mathematical structure of differential equations. There are a few fundamental results and various numerical techniques appropriate to different structural properties. These are discussed in the following sections.

3 Analytical and Numerical Methods: An Overview

The problem of solving Maxwell's equations in a one-dimensional photonic crystal is, at its core, a boundary value problem for a second-order ordinary differential equation with periodic coefficients. There are various distinct methods to solve this mathematical problem. These methods have been developed because of computational limitations of

different time-periods, and the fact that each method is optimised for a different physical question. A method well suited for computing a property, say reflectance spectrum, may be poorly suited for computing the other property, say the band structure. Similarly, different types of structures of the photonic crystal require different approach to solve these equations. Thus, the method that works successfully for an infinite periodic crystal may fail numerically for a thick finite one. In this section, a brief introduction of each of the following methods is given within its historical context.

1. The *transfer matrix method* (**TMM**) involves representing each layer of a multilayer structure by a single 2×2 matrix and then simply multiplying these matrices in sequence to find out the optical response. It is very efficient for one-dimensional systems with arbitrary layer sequences, though this direct multiplication can become numerically unstable for very thick structures.
2. The *scattering matrix method* (**SMM**) avoids numerical instabilities in thick structures. It describes each layer by a matrix that relates incoming to outgoing wave amplitudes, rather than propagating total fields.
3. The *finite-difference time-domain method* (**FDTD**) solves Maxwell's equations directly in time domain by dividing space and time into a fine grid and advancing Maxwell's curl equations step by step in time, making it naturally suited for dispersive, nonlinear, and time-resolved problems.
4. The *plane wave expansion method* (**PWEM**) expands fields and dielectric functions in Fourier series, providing systematic band structure calculations particularly suited for infinite periodic systems.
5. *Coupled mode theory* (**CMT**) describes the photonic crystal not by solving the wave equation directly but by writing equations for how the amplitudes of known modes exchange energy, making it particularly suited for defect cavities and weakly perturbed periodic structures.
6. *Effective medium theory* (**EMT**) is included here more for completeness of this description and it is not suited for studying the characteristic bandgap formation of photonic crystals. It describes the long-wavelength behaviour of layered media ($\Lambda \ll \lambda$). In this regime, the periodic structure ceases to behave as a photonic crystal.

These six methods have emerged to answer different questions and explore different geometries. TMM attempts to find out what the R and T are for this finite stack. SMM tries to compute R and T stably when TMM overflows. FDTD becomes the most convenient tool when it is to be found out how a pulse propagates through this structure

Table 2: Historical development and scope of methods for one-dimensional photonic crystals.

Method	Originator	Year	Primary use
TMM	Abelès	1950	R/T spectra, finite structures
SMM	Ko and Inkson	1988	Thick/absorbing structures
FDTD	Yee	1966	Time domain, nonlinear media
PWE	Ho, Chan, Soukoulis	1990	Band structure
CMT	Pierce; Haus	1954; 1991	Defects, coupled cavities
EMT	Rayleigh	1892	Long-wavelength limit

The originating works for TMM, SMM, FDTD and PWE are cited in the text as [20], [28], [21] and [9] respectively. The coupled-mode and effective-medium entries are given for orientation only; neither method is treated in this review.

in time. PWE is very popular technique and helps find out the band structure of this infinite crystal. CMT finds application in exploring the defect modes and interaction between propagating modes. EMT is used to study the long-wavelength behaviour.

Three methods, viz., TMM, SMM and FDTD are frequently used where the nature of periodicity of the unit cell is not exploited to find a solution. This paper deals with these three methods in the following subsections. The mathematical foundations of the remaining three methods will be dealt with in another paper.

3.1 Transfer Matrix Method

The earliest systematic formulation of transfer matrix method (TMM), the most widely used analytical technique for solving electromagnetic problems in one-dimensional stratified media [19], is due to **Abelès (1950)**, who showed that the reflection and transmission of a multilayer stack could be computed by multiplying 2×2 matrices, one per layer [20]. The method is exact for any sequence of plane parallel layers and is computationally inexpensive. It became the standard tool of the optical coating industry decades before the term photonic crystal was coined. Its application to photonic crystals is described in detail in the graduate textbook by **Yeh (1988)** [19]. The TMM has been proved to be suitable for computing reflectance and transmittance spectra of finite structures, as well as for deriving the photonic band structure of infinite periodic structures through Bloch's theorem. Its principal limitation is numerical instability for very thick structures or highly absorbing media, where matrix elements grow exponentially and cause overflow.

3.1.1 Formulation for a Single Interface

Consider a plane wave incident on an interface between two semi-infinite media with refractive indices n_1 and n_2 . For TE polarization at normal incidence, the electric field in medium 1 (incident and reflected waves) is:

$$E_1(z) = Ae^{ik_1z} + Be^{-ik_1z} \quad (19)$$

where $k_1 = n_1k_0$, A is the incident amplitude, and B is the reflected amplitude. In medium 2 (transmitted wave):

$$E_2(z) = Ce^{ik_2z} \quad (20)$$

where $k_2 = n_2k_0$ and C is the transmitted amplitude.

Applying boundary conditions at $z = 0$ (continuity of E and dE/dz):

$$A + B = C \quad (21)$$

$$n_1(A - B) = n_2C \quad (22)$$

Solving these equations yields the Fresnel coefficients:

$$r_{12} = \frac{B}{A} = \frac{n_1 - n_2}{n_1 + n_2} \quad (23)$$

$$t_{12} = \frac{C}{A} = \frac{2n_1}{n_1 + n_2} \quad (24)$$

3.1.2 Single Layer Transfer Matrix

For a single layer of thickness d and refractive index n , we propagate the fields from the entrance face ($z = 0$) to the exit face ($z = d$). We work with the state vector (E, H) , where E is the tangential electric field and H is the tangential magnetic field. For TE polarization and a monochromatic wave of angular frequency ω , the general solution inside the layer is:

$$E(z) = A_+ e^{ink_0z} + A_- e^{-ink_0z}, \quad (25)$$

where $k_0 = \omega/c$ is the free-space wave number, so that nk_0 is the wave number inside the layer. The magnetic field follows from Faraday's law,

$$\mu_0 \frac{\partial H_y}{\partial z} = -\frac{\partial E_x}{\partial t} \quad (26)$$

giving:

$$H(z) = n (A_+ e^{ink_0z} - A_- e^{-ink_0z}), \quad (27)$$

where we have used the plane wave relation $H = nE/\eta_0$, with $\eta_0 = \sqrt{\mu_0/\epsilon_0} \approx 377 \, \Omega$ the impedance of free space, and have adopted the convention of absorbing η_0 into the definition of H so that E and H carry the same dimensions in the transfer matrix.

Evaluating at $z = 0$ and $z = d$:

$$E(0) = A_+ + A_-, \quad H(0) = n(A_+ - A_-), \quad (28)$$

$$E(d) = A_+ e^{i\delta} + A_- e^{-i\delta}, \quad H(d) = n(A_+ e^{i\delta} - A_- e^{-i\delta}), \quad (29)$$

where $\delta \equiv nk_0 d$ is the *optical phase thickness* of the layer. Eliminating A_+ and A_- gives:

$$E(d) = E(0) \cos \delta + \frac{i}{n} H(0) \sin \delta, \quad (30)$$

$$H(d) = in E(0) \sin \delta + H(0) \cos \delta. \quad (31)$$

In matrix form:

$$\boxed{\begin{pmatrix} E(d) \\ H(d) \end{pmatrix} = \underbrace{\begin{pmatrix} \cos \delta & \frac{i}{n} \sin \delta \\ in \sin \delta & \cos \delta \end{pmatrix}}_{\mathbf{M}} \begin{pmatrix} E(0) \\ H(0) \end{pmatrix}}. \quad (32)$$

This 2×2 matrix $\mathbf{M}$ is the *transfer matrix* for a single layer. Two properties are immediate:

1. $\det(\mathbf{M}) = \cos^2 \delta + \sin^2 \delta = 1$: the matrix is *unimodular*, reflecting lossless propagation.
2. $\mathbf{M}(\delta = 0) = \mathbf{I}$: a layer of zero thickness leaves the fields unchanged.

A note on conventions: some treatments use the state vector (E, E') with $E' = dE/dz$. That convention yields a real transfer matrix with a minus sign in position (2,1). The (E, H) convention used here, standard in thin-film optics (Macleod 2010), produces a complex matrix but one that connects more directly to the physical observables of reflection and transmission. The two conventions are related by $H = (-i/k_0) E'$ and give identical physics; only the matrix elements differ.

For one-dimensional structures with interfaces perpendicular to the z -axis, the above conditions reduce to continuity of the tangential field components. For the two polarizations this means:

$$\text{TE: } E_x \text{ and } H_y \text{ are continuous.} \quad (33)$$

$$\text{TM: } H_x \text{ and } E_y \text{ are continuous.} \quad (34)$$

Equivalently, using the field equations to express H_y and E_y in terms of derivatives:

$$\text{TE: } E_x \text{ and } \frac{1}{\mu_r} \frac{dE_x}{dz} \text{ are continuous.} \quad (35)$$

$$\text{TM: } H_x \text{ and } \frac{1}{\epsilon_r} \frac{dH_x}{dz} \text{ are continuous.} \quad (36)$$

For the non-magnetic dielectric materials considered throughout this review ($\mu_r = 1$), Eq. (35) simplifies to continuity of E_x and dE_x/dz . The factor $1/\mu_r = 1$ disappears, but it is important to carry it through the general case: the asymmetry between TE and TM (an explicit ϵ_r factor in TM but not in TE) is not an inconsistency but a direct consequence of the vector nature of Maxwell's equations and the different roles played by ϵ_r and μ_r in each polarization. It is precisely this asymmetry that causes TE and TM bandgaps to open at different frequencies in the same structure at oblique incidence.

These boundary conditions are essential for constructing solutions to the wave equation in multilayer structures, as discussed in the following sections.

3.1.3 Multilayer Structures

For a multilayer structure consisting of N layers, the total transfer matrix is the product of individual layer matrices:

$$\mathbf{M}_{\text{total}} = \mathbf{M}_1 \mathbf{M}_2 \cdots \mathbf{M}_{N-1} \mathbf{M}_N \quad (37)$$

Note that matrix multiplication is non-commutative, and the order matters: $\mathbf{M}_1$ describes the layer nearest the incident medium, so it appears leftmost in the product.

Following the thin-film convention of Macleod [17], the total transfer matrix relates the entrance fields (E_0, H_0) to the exit fields (E_N, H_N) by:

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} E_N \\ H_N \end{pmatrix} \quad (38)$$

where M_{11} , M_{12} , M_{21} , M_{22} are the elements of $\mathbf{M}_{\text{total}}$, obtained by explicit matrix multiplication.

In order to derive expressions for r and t , consider the structure embedded between a semi-infinite incident medium of refractive index n_0 and a semi-infinite substrate of refractive index n_s . In the incident medium, the total field is a superposition of a forward-propagating incident wave of amplitude 1 (normalised) and a backward-propagating re-

flected wave of amplitude r :

$$E_{\text{in}} = 1 + r, \quad (39)$$

$$H_{\text{in}} = n_0(1 - r). \quad (40)$$

The minus sign in H_{in} arises because reversing the direction of propagation reverses the magnetic field but not the electric field: for a wave travelling in the $-z$ direction, $H = -nE/\eta_0$, whereas for a wave travelling in the $+z$ direction, $H = +nE/\eta_0$.

In the substrate, only a forward-propagating transmitted wave of amplitude t exists:

$$E_{\text{out}} = t, \quad (41)$$

$$H_{\text{out}} = n_s t. \quad (42)$$

Applying the transfer matrix with entrance fields on the left:

$$1 + r = (M_{11} + M_{12}n_s)t, \quad (43)$$

$$n_0(1 - r) = (M_{21} + M_{22}n_s)t. \quad (44)$$

Dividing (44) by (43) eliminates t :

$$\frac{n_0(1 - r)}{1 + r} = \frac{M_{21} + M_{22}n_s}{M_{11} + M_{12}n_s} \quad (45)$$

Let $P = n_0(M_{11} + M_{12}n_s)$ and $Q = (M_{21} + M_{22}n_s)$. Cross-multiplying:

$$P(1 - r) = Q(1 + r) \implies P - Q = r(P + Q) \quad (46)$$

Solving for r :

$$r = \frac{n_0(M_{11} + M_{12}n_s) - (M_{21} + M_{22}n_s)}{n_0(M_{11} + M_{12}n_s) + (M_{21} + M_{22}n_s)} \quad (47)$$

From (43):

$$t = \frac{1 + r}{M_{11} + M_{12}n_s} \quad (48)$$

Substituting $1 + r = \frac{2P}{P + Q}$:

$$t = \frac{2n_0}{n_0(M_{11} + M_{12}n_s) + (M_{21} + M_{22}n_s)} \quad (49)$$

The reflectance $R = |r|^2$ and transmittance $T = (n_s/n_0)|t|^2$ satisfy energy conservation $R + T = 1$ in the absence of absorption. For a plane wave in a non-magnetic medium, the time-averaged Poynting vector is proportional to $n|E|^2$, not to $|E|^2$ alone, which accounts

for the factor n_s/n_0 in T .

3.1.4 Numerical Implementation

The transfer matrix method is straightforward to implement numerically. The basic algorithm for computing the reflectance spectrum of an N -period structure is:

1. Initialize the total transfer matrix as the identity matrix.
2. For each period:
 - (a) For each layer in the unit cell:
 - i. Compute the optical phase thickness $\delta = nk_0d$
 - ii. Construct the layer transfer matrix
 - iii. Multiply it with the accumulated matrix
3. Extract matrix elements and compute reflection coefficient
4. Calculate reflectance $R = |r|^2$

For band structure calculations, one varies the frequency and solves for the Bloch wave vector from the trace condition.

A key numerical consideration is the accumulation of floating-point errors in long structures. For structures with many periods, the eigenvalue method or scattering matrix formalism may be more numerically stable [30].

3.1.5 Worked Example: SiO₂/TiO₂ Quarter-Wave Stack at 550 nm

We demonstrate the TMM with a concrete calculation. Consider a five-period binary photonic crystal of alternating TiO₂ ($n_H = 2.30$) and SiO₂ ($n_L = 1.46$) layers, designed for a central wavelength $\lambda_0 = 550$ nm, embedded between air ($n_0 = 1.0$) and glass ($n_s = 1.52$). The unit cell is taken in the order HL, so that the high-index TiO₂ layer faces the incident medium; this is the standard quarter-wave mirror arrangement and gives the largest reflectance for a given number of periods.

Step 1: Layer Thicknesses.

$$d_H = \frac{550}{4 \times 2.30} = 59.78 \text{ nm} \quad (50)$$

$$d_L = \frac{550}{4 \times 1.46} = 94.18 \text{ nm} \quad (51)$$

Step 2: Phase Thicknesses at λ_0 .

At the design wavelength, $k_0 = 2\pi/550 \text{ nm}^{-1}$:

$$\delta_H = n_H k_0 d_H = \frac{\pi}{2} \quad (52)$$

$$\delta_L = n_L k_0 d_L = \frac{\pi}{2} \quad (53)$$

Both layers are exactly quarter-wave thick at λ_0 , as designed.

Step 3: Individual Transfer Matrices at λ_0 .

Since $\delta_H = \delta_L = \pi/2$, we have $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$:

$$\mathbf{M}_H = \begin{pmatrix} 0 & \frac{i}{n_H} \\ in_H & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{i}{2.30} \\ 2.30i & 0 \end{pmatrix} \quad (54)$$

$$\mathbf{M}_L = \begin{pmatrix} 0 & \frac{i}{n_L} \\ in_L & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{i}{1.46} \\ 1.46i & 0 \end{pmatrix} \quad (55)$$

Step 4: Unit Cell Transfer Matrix.

$$\mathbf{M}_{\text{cell}} = \mathbf{M}_H \mathbf{M}_L = \begin{pmatrix} -\frac{n_L}{n_H} & 0 \\ 0 & -\frac{n_H}{n_L} \end{pmatrix} = \begin{pmatrix} -0.635 & 0 \\ 0 & -1.575 \end{pmatrix} \quad (56)$$

Step 5: Bloch Condition.

$$\cos(K\Lambda) = \frac{1}{2} \text{Tr}(\mathbf{M}_{\text{cell}}) = \frac{-0.635 + (-1.575)}{2} = -1.105 \quad (57)$$

Since $|\cos(K\Lambda)| = 1.105 > 1$, the design wavelength lies *inside* the photonic bandgap. The quarter-wave condition maximises $|\text{Tr}(\mathbf{M}_{\text{cell}})|$, so $\lambda_0 = 550 \text{ nm}$ also sits at the centre of the gap.

Step 6: Peak Reflectance of Five-Period Stack. For N periods the total matrix is the N th power of the unit cell. Since $\mathbf{M}_{\text{cell}}$ is diagonal, so is every power of it, and for $N = 5$:

$$\mathbf{M}_{\text{total}} = (\mathbf{M}_H \mathbf{M}_L)^5 = \begin{pmatrix} -\left(\frac{n_L}{n_H}\right)^5 & 0 \\ 0 & -\left(\frac{n_H}{n_L}\right)^5 \end{pmatrix} \quad (58)$$

With $M_{12} = M_{21} = 0$, Eq. (47) reduces to a ratio of the two diagonal elements alone. Dividing M_{11} by M_{22} turns the two fifth powers into a single tenth power, so the exponent below is $2N = 10$:

$$R_5 = \left[\frac{1 - \frac{n_0}{n_s} \left(\frac{1.46}{2.30}\right)^{10}}{1 + \frac{n_0}{n_s} \left(\frac{1.46}{2.30}\right)^{10}} \right]^2 = \left[\frac{1 - 0.00699}{1 + 0.00699} \right]^2 \approx 0.972 \quad (59)$$

A five-period stack, with the high-index layer facing the incident medium, already achieves 97.2% reflectance. Ten periods would exceed 99.9%.

Step 7: Bandgap Width.

$$\frac{\Delta\omega}{\omega_0} = \frac{4}{\pi} \sin^{-1} \left(\frac{2.30 - 1.46}{2.30 + 1.46} \right) = \frac{4}{\pi} \sin^{-1}(0.2234) \approx 0.287 \quad (60)$$

The bandgap spans approximately 28.7% of the central frequency, corresponding to wavelengths from roughly 481 nm to 642 nm—covering the visible range from blue-green through to orange-red.

Summary Table.

Quantity	Formula	Value
d_H	$\lambda_0/4n_H$	59.78 nm
d_L	$\lambda_0/4n_L$	94.18 nm
Period Λ	$d_H + d_L$	153.96 nm
$\text{Tr}(\mathbf{M}_{\text{cell}})$	—	−2.210
R_5	Eq. (59)	97.2%
$\Delta\omega/\omega_0$	Eq. (60)	28.7%

Table 3: Comparison of computational methods for 1D photonic crystals.

Feature	TMM	PWE	FDTD	SMM
Finite structures	✓	×	✓	✓
Band structure	✓	✓	Indirect	✓
Nonlinear/dispersive	×	×	✓	×
Numerical stability	Medium	High	High	High
Implementation effort	Low	Medium	High	Medium
Broadband in one run	×	×	✓	×

A few exercises of pedagogical relevance are given here which can help the reader to learn the idea in a better way:

1. Repeat the worked example for ZnS/MgF₂ ($n_H = 2.35$, $n_L = 1.38$) at $\lambda_0 = 633$ nm, on the same air/glass configuration with the high-index layer facing the air. How many periods are needed to reach $R > 99.5\%$?
2. Show that for lossless media every layer matrix has the form $\begin{pmatrix} a & ib \\ ic & d \end{pmatrix}$ with a, b, c, d real, that this form is closed under multiplication, and hence that $\text{Tr}(\mathbf{M}_{\text{cell}})$ is always real. Show further that a symmetric unit cell (LHL) additionally gives $M_{11} = M_{22}$.
3. Write a MATLAB or Python script to plot the reflectance spectrum of the SiO₂/TiO₂ stack from 400 nm to 800 nm using the TMM derived in this section.

3.2 Scattering Matrix Method

A brief history of the method. The scattering matrix reached photonic crystals from an unexpected direction. Whittaker and Culshaw adapted the S -matrix formalism, originally built by Ko and Inkson for electron tunneling through semiconductor heterostructures [28], to compute reflectivity and emission spectra of patterned photonic-crystal waveguides [29]. The motivation was practical: the ordinary transfer matrix tracks the total field in each layer, and for the deep, wavelength-scale gratings typical of photonic crystals this overflows numerically once evanescent modes are present. Busch et al., in their review of periodic nanostructures for photonics, note that what the diffractive-optics community calls rigorous coupled-wave analysis is, in the photonic-crystal literature, generally known by this name — the scattering-matrix approach — citing Whittaker and Culshaw among its defining references [31].

3.2.1 Formulation

For a single layer or a block of layers, the scattering matrix $\mathbf{S}$ relates incoming amplitudes (a^+, b^-) to outgoing amplitudes (a^-, b^+) :

$$\begin{pmatrix} a^- \\ b^+ \end{pmatrix} = \underbrace{\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}}_{\mathbf{S}} \begin{pmatrix} a^+ \\ b^- \end{pmatrix}, \quad (61)$$

where superscripts $+$ and $-$ denote waves propagating in the $+z$ and $-z$ directions respectively, and a , b refer to the left and right sides of the structure. The diagonal blocks S_{11} and S_{22} are reflection coefficients; the off-diagonal blocks S_{12} and S_{21} are transmission coefficients.

Two scattering matrices $\mathbf{S}^{(1)}$ and $\mathbf{S}^{(2)}$ for adjacent blocks are combined using the *Redheffer star product* $\star$ [29, 30]:

$$\begin{aligned} (\mathbf{S}^{(1)} \star \mathbf{S}^{(2)})_{11} &= S_{11}^{(1)} + S_{12}^{(1)} \left(\mathbf{I} - S_{11}^{(2)} S_{22}^{(1)} \right)^{-1} S_{11}^{(2)} S_{21}^{(1)}, \\ (\mathbf{S}^{(1)} \star \mathbf{S}^{(2)})_{12} &= S_{12}^{(1)} \left(\mathbf{I} - S_{11}^{(2)} S_{22}^{(1)} \right)^{-1} S_{12}^{(2)}, \\ (\mathbf{S}^{(1)} \star \mathbf{S}^{(2)})_{21} &= S_{21}^{(2)} \left(\mathbf{I} - S_{22}^{(1)} S_{11}^{(2)} \right)^{-1} S_{21}^{(1)}, \\ (\mathbf{S}^{(1)} \star \mathbf{S}^{(2)})_{22} &= S_{22}^{(2)} + S_{21}^{(2)} \left(\mathbf{I} - S_{22}^{(1)} S_{11}^{(2)} \right)^{-1} S_{22}^{(1)} S_{12}^{(2)}. \end{aligned} \quad (62)$$

Unlike ordinary matrix multiplication, the star product never requires inverting the matrix $\hat{q}$ that carries the evanescent phase factors directly [29]: since $|\hat{q}_{nn}| < 1$ for $\text{Im}(q_n) > 0$, the growing exponentials that afflict the TMM for thick structures never appear.

3.2.2 Relation to TMM and Practical Use

For thin structures and moderate refractive index contrast, the TMM and SMM give identical results to numerical precision. The SMM becomes necessary once evanescent fields inside a bandgap grow large enough that the TMM's total-field matrix entries lose precision to floating-point cancellation — roughly when $e^{2\alpha N\Lambda} \gg 1$, where α is the evanescent decay constant inside the bandgap, N the number of periods, and Λ the period. A quantitative threshold in terms of N and index contrast depends on the working precision and on the details of the implementation, and is best determined empirically for a given structure. For band structure calculations the two methods are equivalent, since the Bloch condition is applied to a single unit cell where no instability arises.

3.3 Finite-Difference Time-Domain Method

FDTD is a popular technique of computational electromagnetics to solve Maxwell's equations numerically. This was proposed by **Yee (1966)** [21]. Space is cut into a grid, finite in all three directions. Time is cut into finite steps. A continuous world becomes a lattice. This is true of any finite-difference scheme. Yee proposed to place $\mathbf{E}$ and $\mathbf{H}$ on grids offset from each other by half a cell in space, and to advance them by half a time-step in time, each one staggering over the other. It advances in time, one small step after another, replacing every derivative, both in space, and in time, by a ratio of the difference. In this scheme, $\mathbf{H}$ is known at integer time steps $(\dots(n-1), n, (n+1), \dots)$ and half-integer space points $(\dots(i-1/2), (i+1/2), \dots)$; $\mathbf{E}$ is known at half-integer time steps $(\dots(n-1/2), (n+1/2), \dots)$ and integer space points $(\dots(i-1), i, (i+1), \dots)$.

To elaborate further, consider a time derivative of $\mathbf{E}$, at some fixed spatial grid point i . It is approximated by the difference between the field's value at time step $n + \frac{1}{2}$ and at time step $n - \frac{1}{2}$, divided by Δt . Therefore, for TE polarization ($\mathbf{E} = E_x \hat{\mathbf{x}}$, $\mathbf{H} = H_y \hat{\mathbf{y}}$), we can write:

$$\frac{\partial E_x}{\partial t} = \frac{E_x^{n+1/2}(i) - E_x^{n-1/2}(i)}{\Delta t} \quad (63)$$

Similarly, consider a time derivative, at some fixed spatial grid point $i + \frac{1}{2}$. It is approximated by the difference between the field's value at time step $n + 1$ and at time step n , divided by Δt .

$$\frac{\partial H_y}{\partial t} = \frac{H_y^{n+1}(i + \frac{1}{2}) - H_y^n(i + \frac{1}{2})}{\Delta t} \quad (64)$$

Now consider a space derivative. Suppose we need the space derivative of $E_x \hat{\mathbf{x}}$, at some fixed instant $n + \frac{1}{2}$. It is approximated by the difference between the field's value at grid

point $i + 1$ and at grid point i , divided by Δz .

$$\frac{\partial E_x}{\partial z} = \frac{E_x^{n+1/2}(i+1) - E_x^{n+1/2}(i)}{\Delta z} \quad (65)$$

Similarly, to take the space derivative of $H_y \hat{\mathbf{y}}$, at some fixed instant n , it is approximated by the difference between the field's value at grid point $i + \frac{1}{2}$ and at grid point $i - \frac{1}{2}$, divided by Δz .

$$\frac{\partial H_y}{\partial z} = \frac{H_y^n(i + \frac{1}{2}) - H_y^n(i - \frac{1}{2})}{\Delta z} \quad (66)$$

Neither field ever needs to be evaluated where it isn't already naturally defined by the very equation that updates it. This one placement choice is what turns a crude difference scheme into a stable, energy-respecting algorithm. This is the uniqueness of the *Yee's algorithm*. For TE polarization ($\mathbf{E} = E_x \hat{\mathbf{x}}$, $\mathbf{H} = H_y \hat{\mathbf{y}}$, propagation along z), Faraday's and Ampère's laws reduce to two coupled scalar equations:

$$\frac{E_x^{n+1/2}(i) - E_x^{n-1/2}(i)}{\Delta t} = -\frac{1}{\epsilon_0 \epsilon_r(i)} \frac{H_y^n(i + \frac{1}{2}) - H_y^n(i - \frac{1}{2})}{\Delta z} \quad (67)$$

$$\frac{H_y^{n+1}(i + \frac{1}{2}) - H_y^n(i + \frac{1}{2})}{\Delta t} = -\frac{1}{\mu_0} \frac{E_x^{n+1/2}(i+1) - E_x^{n+1/2}(i)}{\Delta z} \quad (68)$$

where i indexes spatial grid points, n indexes time steps, Δz is the spatial step, and Δt the time step. Read Eq. (67): E_x at the half-time-step ahead is built from H_y at the two neighbouring half-space-points, one step back in time. Read Eq. (68): H_y at the next integer time step is built from E_x at the two neighbouring integer space points, half a step back. Each field updates the other; neither ever updates itself. The scheme is only conditionally stable: the time step must satisfy the Courant–Friedrichs–Lewy (CFL) condition,

$$\Delta t \leq \frac{\Delta z}{c} \quad (69)$$

[22], or the simulation diverges. For spatial accuracy, a rule of thumb is $\Delta z \leq \lambda_{\min}/20$, with $\lambda_{\min}$ the smallest wavelength in the highest-index region. Strict boundary conditions are imposed for physically consistent solutions and simulations.

3.3.1 Applications to Photonic Crystals

FDTD predates photonic crystals by two decades. Yee was solving radar scattering problems: complicated three-dimensional shapes, no symmetry to exploit, no analytical method in sight. The method was carried to full maturity by **Taflove and Hagness** [22]. Its later adoption by the photonic-crystal community was almost inevitable:

FDTD assumes nothing about geometry or periodicity, handles dispersive, nonlinear, and anisotropic media without complaint, and computes a broadband response from a single simulation, by exciting the structure with a short pulse and Fourier-transforming the time-domain output.

Several variants of FDTD exist like alternating-direction-implicit (ADI-FDTD), pseudo-spectral time-domain (PSTD), multiresolution time-domain (MRTD), etc. FDTD is at its best computing transmission and reflection spectra, field distributions, and the quality factors of resonances. Band structures are not its natural output. band structures require Fourier post-processing of the time-domain response, an extra step frequency-domain methods like PWE skip entirely. Where FDTD earns its computational cost is in nonlinear effects, ultrafast dynamics, and irregular geometries.

4 Types of One-Dimensional Photonic Crystals

One-dimensional photonic crystals are classified by the arrangement of their dielectric layers. The simplest periodic arrangements use two or more materials repeating with a fixed unit cell. Quasiperiodic arrangements follow deterministic but non-repeating sequences. Structurally modified designs introduce deliberate variations along the growth direction. Each class offers distinct optical properties unavailable in the others.

Table 4: Classification of one-dimensional photonic crystals.

Type	Key early work	Key property	Application
Binary	Rayleigh, 1887	Wide bandgap	DBR mirrors, VCSELs
Ternary	Awasthi <i>et al.</i> , 2006	Multiple bandgaps	Multi-wavelength filters
Quaternary	—	Flat-top bandgap	Narrowband filters
Fibonacci	Kohmoto <i>et al.</i> , 1987	Fractal spectrum	Multi-frequency filtering
Thue-Morse	Axel & Peyrière, 1989	Broadband reflection	Broadband mirrors
Cantor	Monsoriu <i>et al.</i>	Self-similar gaps	Hierarchical filtering
Defect-mode	Yablonovitch <i>et al.</i> , 1991	Localised state in gap	Microcavity lasers
Chirped	Szipöcs <i>et al.</i> , 1994	Broadband dispersion	Ultrafast optics
Rugate	Southwell, 1989	No sidelobes	Clean spectral filters

The quasiperiodic entries follow the survey of aperiodic photonic structures in [23].

4.1 Periodic Structures

4.1.1 Binary Photonic Crystals

Binary photonic crystals consist of alternating layers of two dielectric materials with refractive indices n_H and n_L and thicknesses d_H and d_L , repeating with period $\Lambda = d_H + d_L$. The concept of a photonic bandgap in such structures is not a new one and it was Lord Rayleigh (1887) [1], who showed that stratified media with periodic refractive

index variations reflect light over a range of frequencies. The quarter-wave stack, where $n_H d_H = n_L d_L = \lambda_0/4$, maximises the reflectance at the design wavelength λ_0 and was developed extensively in the optical thin-film community by **Macleod** and others [17]. **Yablonovitch (1987)** and **John (1987)** [7, 8] developed and established the modern photonic crystal framework which unify these structures with higher-dimensional periodic systems. Binary structures remain the most widely fabricated and studied type, serving as distributed Bragg reflectors in Vertical-Cavity Surface-Emitting Laser (VCSELs), high-reflectivity mirrors, and wavelength-selective filters.

4.1.2 Ternary Photonic Crystals

Ternary photonic crystals incorporate three distinct dielectric materials in the unit cell, with refractive indices n_1, n_2, n_3 and corresponding thicknesses d_1, d_2, d_3 . The additional degree of freedom allows simultaneous control over bandgap width, position, and passband transmittance, which binary structures cannot independently optimise. The gradual impedance matching achievable with a third material reduces interface reflections in the passbands, yielding higher transmittance than binary designs of comparable period. Systematic study of ternary one-dimensional photonic crystals was reported by **Awasthi, Malaviya and Ojha (2006)** [18], who showed that the third material widens the omnidirectional total-reflection range beyond what a binary stack of the same materials can achieve. Ternary configurations can also produce multiple simultaneous bandgaps at harmonically unrelated frequencies, a feature exploited in multi-wavelength filtering applications.

4.1.3 Quaternary Photonic Crystals

Quaternary photonic crystals extend the unit cell to four distinct layers. The further increase in free parameters allows engineering of flat-top bandgaps, suppression of side-lobes, and fine control over defect-mode frequencies. Quaternary configurations have been examined for omnidirectional reflection and optical filtering, as a natural extension of the ternary designs discussed above [18]. The fabrication complexity increases with the number of distinct materials, but modern atomic layer deposition techniques have made quaternary designs practically accessible.

4.2 Quasiperiodic Structures

Quasiperiodic structures are the structures which have an order which is between perfect periodic and disordered. They are generated by deterministic substitution rules, possess long-range correlations but no translational periodicity, and exhibit photonic spectra that are neither the discrete bandgaps of periodic crystals nor the featureless spectra of random media. Their study was motivated by the discovery of quasicrystals by **Shechtman et al.**

(1984) in condensed matter physics, which prompted the question of whether analogous photonic structures would exhibit unusual optical properties. The aperiodic photonic structures described in the following subsections are surveyed comprehensively by Maciá [23]; the one-dimensional case in particular is treated at length by Ghulinyan [24].

4.2.1 Fibonacci Photonic Crystals

Fibonacci photonic crystals are generated by the substitution rule $A \rightarrow AB$, $B \rightarrow A$, producing the sequence $A, AB, ABA, ABAAB, \dots$ where A and B represent layers of two different dielectric materials. The number of layers in successive generations follows the Fibonacci series, and the ratio of layer counts converges to the golden ratio $\phi = (1+\sqrt{5})/2$. Quasiperiodic Fibonacci semiconductor superlattices were first realised experimentally by **Merlin et al. (1985)** [23,24], thus predating the photonic crystal concept itself. It is to be remembered that their original motivation was the electronic and vibrational (Raman-scattering) properties of the quasiperiodic structure, perhaps not its photonic bandgap behaviour. The specifically optical properties of such quasiperiodic dielectric sequences were investigated shortly after by **Kohmoto, Sutherland, and Iguchi (1987)**, who showed that the transmission spectrum exhibits a self-similar, fractal-like structure and that field intensities can be dramatically enhanced at specific frequencies [14]. Unlike periodic crystals, Fibonacci structures do not possess a true bandgap but rather a spectrum of allowed and forbidden frequency regions, with self-similarity at all scales. This property has been exploited for multi-frequency filtering and for studying localisation phenomena in deterministic aperiodic systems.

4.2.2 Thue-Morse Photonic Crystals

The Thue-Morse sequence is generated by the substitution rule $A \rightarrow AB$, $B \rightarrow BA$, producing $A, AB, ABBA, ABBABAAB, \dots$. It differs from the Fibonacci sequence in that it possesses a higher degree of symmetry: the sequence is invariant under the simultaneous exchange $A \leftrightarrow B$ and reversal, a property that holds for the even generations of the sequence. **Axel and Peyrière (1989)** analysed the spectral properties of Thue-Morse sequences [23]. In photonics, Thue-Morse structures have been found to exhibit broader effective reflection bands than periodic structures of the same total thickness. This property has made them attractive for broadband mirror applications.

4.2.3 Cantor Photonic Crystals

Cantor photonic crystals are constructed using the triadic Cantor set: a unit interval is divided into three equal parts and the middle segment is replaced by a low-index material, with the rule applied recursively to the remaining high-index segments. The resulting structure is self-similar at all scales and possesses a fractal dimension $\log 2 / \log 3 \approx 0.631$.

Monsoriu and co-workers systematically studied the transmission spectra of Cantor photonic crystals [23] and demonstrated that the fractal geometry produces a transmission spectrum that mirrors the self-similar character of the structure itself, with bandgaps nested within bandgaps at successive length scales. This hierarchical gap structure cannot be achieved by any periodic design.

4.3 Structurally Modified Periodic Structures

4.3.1 Defect-Mode Photonic Crystals

Introducing a single layer with different thickness or refractive index into an otherwise periodic binary structure breaks the translational symmetry and creates a localised electromagnetic state within the photonic bandgap. The frequency of this defect mode depends on the optical thickness of the defect layer. This allows precise wavelength selection. The existence of such donor and acceptor modes in photonic bandgap structures, in direct analogy with impurity states in semiconductor band theory, was demonstrated theoretically and experimentally by **Yablonovitch, Gmitter, Meade, Rappe, Brommer, and Joannopoulos (1991)** [11]. Defect-mode one-dimensional photonic crystals function as narrow-band transmission filters with quality factors determined by the reflectance of the surrounding periodic mirrors. They form the basis of Fabry-Pérot microcavities, distributed feedback lasers, and wavelength-division multiplexing components.

4.3.2 Chirped Photonic Crystals

In a chirped photonic crystal, the period Λ varies monotonically along the growth direction, typically increasing or decreasing linearly. Each local region of the structure reflects a different wavelength, producing a broadband reflection response unavailable from uniform periodic structures. **Szipöcs et al. (1994)** introduced chirped multilayer mirrors for compensation of group-velocity dispersion in ultrafast laser systems [17], demonstrating that the chirped design could compress femtosecond pulses by pre-compensating the dispersion accumulated in optical components. Chirped designs are now standard components in mode-locked laser cavities and optical coherence tomography systems.

4.3.3 Rugate Filters

Rugate filters replace the discrete layer interfaces of standard multilayers with a continuously and smoothly varying refractive index profile, typically sinusoidal:

$$n(z) = n_{\text{avg}} + \frac{\Delta n}{2} \sin\left(\frac{2\pi z}{\Lambda}\right) \quad (70)$$

where n_{avg} is the mean index and Δn is the index modulation amplitude. The smooth profile removes the higher harmonics present in a square-wave index profile, which in a conventional quarter-wave stack produce spurious reflection bands at other wavelengths. **Southwell (1989)** [15] further showed that gently tapering the modulation amplitude to zero at the two ends of the structure — rather than switching it on and off abruptly — suppresses the small ripples that otherwise appear near the edges of the main reflectance band.

5 A Very Brief Description of Fabrication Methods and Characterization Methods

Fabrication of one-dimensional photonic crystals is usually done by several ways, prominent of which are thermal evaporation, electron-beam evaporation, sputtering, atomic layer deposition (ALD), sol-gel methods, and pulsed laser deposition. Thermal evaporation is one of the earliest methods. Joule heating is employed to heat and vaporise a source material in a vacuum chamber until it sublimates and condenses onto a substrate. This is suitable for oxides with moderate melting points. Electron-beam evaporation is an improved technique. It focuses a high-energy electron beam onto the source which gets melted and deposited. This is suitable for high-melting-point materials such as TiO_2 ($\sim 1843^\circ\text{C}$) and Ta_2O_5 ($\sim 1872^\circ\text{C}$) that ordinary thermal evaporation cannot vaporize. It is widely used in the optical coating industry [17]. Sputtering employs energetic plasma ions which are bombarded on target atoms, which are ejected and deposited on substrate. This technique offers superior film density and composition control. Atomic layer deposition (ALD) builds films by depositing one molecular monolayer at a time through sequential, self-limiting surface reactions. This provides atomic-scale thickness precision. Sol-gel methods deposit metal-organic precursor solutions by spin- or dip-coating, followed by thermal annealing to form oxide films. This provides a cost-effective route to large-area coatings. Pulsed laser deposition directs short, intense laser pulses onto a solid target. This vaporizes a small amount of this material into a cloud of atoms that settles onto the substrate as a thin film. This is ideal for complex multi-element oxide materials. A comprehensive treatment of all these deposition techniques and the underlying physics of film growth is given in [25].

Films so formed are to be characterized experimentally. This experimental characterization is accomplished using spectrophotometry, ellipsometry, scanning electron microscopy (SEM), X-ray reflectivity, angular-resolved spectroscopy. Spectrophotometry is employed to measure transmittance and reflectance spectra. Ellipsometry helps determine refractive indices and layer thicknesses of individual layers. Scanning electron microscopy (SEM) is used to record cross-sectional images. This helps verify layer thick-

nesses and interface quality. X-ray reflectivity is a technique for high-precision thickness and density measurements. Angular-resolved spectroscopy measures optical response as a function of incident angle.

There are many key experimental achievements which are worth mentioning. These have validated theoretical predictions and enabled applications. For example, the demonstration of omnidirectional reflection by Fink et al. in 1998 used alternating layers of tellurium and polystyrene, and achieved near-unity reflectance over a broad angular range [13]. High-quality-factor optical microcavities based on defects in photonic crystals have achieved Q-factors well in excess of 10^5 , enabling applications in low-threshold lasers and cavity quantum electrodynamics; the highest values reported are for two-dimensional photonic-crystal slabs rather than one-dimensional stacks [26]. Tunable photonic crystals using liquid crystals, electrochromic materials, or mechanical deformation have demonstrated dynamic bandgap control. The integration of periodic dielectric structures with active materials has led to distributed feedback lasers [27] and to light-emitting devices with enhanced extraction efficiency [26].

6 Applications, Challenges and Future Directions

One-dimensional photonic crystals have found widespread use in optical coatings for anti-reflection, high-reflectivity mirrors, and wavelength-selective filters; in lasers as distributed Bragg reflectors in vertical-cavity surface-emitting lasers (VCSELs) and distributed feedback lasers; in sensors that exploit shifts in the bandgap position due to changes in refractive index or layer thickness for chemical and biological detection; in thermal management through wavelength-selective emitters and absorbers for energy applications; and in color generation via structural color as a durable, environmentally friendly alternative to pigments. Current research frontiers include topological photonics, where one-dimensional photonic crystals serve as model systems for exploring topological phases; nonlinear photonic crystals enabling frequency conversion and all-optical switching through engineered dispersion and field enhancement; active and tunable systems integrated with phase-change materials, electro-optic materials, and mechanical actuation; quantum applications that tailor the photonic density of states for controlling spontaneous emission; and machine learning optimization using neural networks and inverse design methods to discover optimal configurations.

Despite this mature understanding and broad applicability, challenges persist in scaling fabrication to large areas with high uniformity, minimizing scattering losses at material interfaces, and integrating diverse functionalities such as tuning, nonlinearity, and gain while preserving structural quality.

Opportunities for advancement lie in leveraging recent progress in nanofabrication, materials science, and computational design, with the convergence of photonic crystals

and emerging fields like metamaterials, metasurfaces, and topological photonics poised to yield novel functionalities, further enhanced by inverse design and machine learning as computational capabilities continue to grow.

7 Conclusion

One-dimensional photonic crystals employ periodic structures to control wave propagation. These have been demonstrated practical utility as well as the elegance of theoretical framework to describe these properties. This review has traced the theoretical foundations, mathematical methods, and computational approaches that underpin our understanding of these systems.

Binary photonic crystals have very simple structure but exhibit rich physics including photonic bandgaps, defect states, and omnidirectional reflection. Ternary structures are useful extension of the stacking and have enabled multiple bandgaps and optimized dispersion relations. The transfer matrix method provides analytical insight, while numerical methods like FDTD enable the study of complex phenomena.

Experimental advances in fabrication and characterization have validated theoretical predictions and enabled applications ranging from optical coatings to quantum technologies. As the field matures, one-dimensional photonic crystals continue to serve as both fundamental test-beds for photonic concepts and practical building blocks for optical devices.

For students entering the field, one-dimensional photonic crystals offer a convenient starting point. The mathematical framework is accessible yet rigorous, physical intuition can be developed through analytical solutions, and computational tools are readily available. The skills and concepts developed in studying one-dimensional systems provide a foundation for understanding higher-dimensional photonic crystals and other periodic optical structure, which are supposed to induce future developments, emphasizing multifunctionality, tunability, and integration with emerging technologies.

References

- [1] Lord Rayleigh. On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure. *Philosophical Magazine*, 24 (147), 145-159, 1887.
- [2] Lord Rayleigh. On the remarkable phenomenon of crystalline reflexion described by Prof. Stokes. *Philosophical Magazine*, 26 (160), 256-265, 1888.
- [3] E. Yablonovitch. Photonic crystals: what's in a name? *Optics and Photonics News*, 18 (3), 12-13, 2007.

- [4] C. G. Darwin. The theory of X-ray reflexion. Part II. *Philosophical Magazine*, 27 (160), 675-690, 1914.
- [5] V. P. Bykov. Spontaneous emission in a periodic structure. *Soviet Physics – JETP*, 35 (2), 269-273, 1972.
- [6] K. Ohtaka. Energy band of photons and low-energy photon diffraction. *Physical Review B*, 19 (10), 5057-5067, 1979.
- [7] E. Yablonovitch. Inhibited spontaneous emission in solid-state physics and electronics. *Physical Review Letters*, 58 (20), 2059-2062, 1987.
- [8] S. John. Strong localization of photons in certain disordered dielectric superlattices. *Physical Review Letters*, 58 (23), 2486-2489, 1987.
- [9] K. M. Ho, C. T. Chan and C. M. Soukoulis. Existence of a photonic gap in periodic dielectric structures. *Physical Review Letters*, 65 (25), 3152-3155, 1990.
- [10] E. Yablonovitch, T. J. Gmitter and K. M. Leung. Photonic band structure: the face-centered-cubic case employing nonspherical atoms. *Physical Review Letters*, 67 (17), 2295-2298, 1991.
- [11] E. Yablonovitch, T. J. Gmitter, R. D. Meade, A. M. Rappe, K. D. Brommer and J. D. Joannopoulos. Donor and acceptor modes in photonic band structure. *Physical Review Letters*, 67 (24), 3380-3383, 1991.
- [12] T. F. Krauss, R. M. De La Rue and S. Brand. Two-dimensional photonic-bandgap structures operating at near-infrared wavelengths. *Nature*, 383 (6602), 699-702, 1996.
- [13] Y. Fink, J. N. Winn, S. Fan, C. Chen, J. Michel, J. D. Joannopoulos and E. L. Thomas. A dielectric omnidirectional reflector. *Science*, 282 (5394), 1679-1682, 1998.
- [14] M. Kohmoto, B. Sutherland and K. Iguchi. Localization of optics: quasiperiodic media. *Physical Review Letters*, 58 (23), 2436-2438, 1987.
- [15] W. H. Southwell. Using apodization functions to reduce sidelobes in rugate filters. *Applied Optics*, 28 (23), 5091-5094, 1989.
- [16] J. D. Joannopoulos, S. G. Johnson, J. N. Winn and R. D. Meade. *Photonic Crystals: Molding the Flow of Light*, Princeton University Press, 2nd Edition, 2008.
- [17] H. A. Macleod. *Thin-Film Optical Filters*, CRC Press, 4th Edition, 2010.
- [18] S. K. Awasthi, U. Malaviya and S. P. Ojha. Enhancement of omnidirectional total-reflection wavelength range by using one-dimensional ternary photonic bandgap material. *Journal of the Optical Society of America B*, 23 (12), 2566-2571, 2006.

- [19] P. Yeh. Optical Waves in Layered Media, Wiley, 1st Edition, 1988.
- [20] F. Abelès. La théorie générale des couches minces. Journal de Physique et le Radium, 11 (7), 307-309, 1950.
- [21] K. S. Yee. Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. IEEE Transactions on Antennas and Propagation, 14 (3), 302-307, 1966.
- [22] A. Taflove and S. C. Hagness. Computational Electrodynamics: The Finite-Difference Time-Domain Method, Artech House, 3rd Edition, 2005.
- [23] E. Maciá. Exploiting aperiodic designs in nanophotonic devices. Reports on Progress in Physics, 75 (3), 036502, 2012.
- [24] M. Ghulinyan. One-dimensional photonic quasicrystals. In: Light Localisation and Lasing, eds. M. Ghulinyan and L. Pavesi, Cambridge University Press, 99-119, 2015. Available as arXiv:1505.02400.
- [25] M. Ohring. Materials Science of Thin Films: Deposition and Structure, Academic Press, 2nd Edition, 2002.
- [26] S. Noda, M. Fujita and T. Asano. Spontaneous-emission control by photonic crystals and nanocavities. Nature Photonics, 1 (8), 449-458, 2007.
- [27] H. Kogelnik and C. V. Shank. Coupled-wave theory of distributed feedback lasers. Journal of Applied Physics, 43 (5), 2327-2335, 1972.
- [28] D. Y. K. Ko and J. C. Inkson. Matrix method for tunneling in heterostructures: Resonant tunneling in multilayer systems. Physical Review B, 38 (14), 9945-9951, 1988.
- [29] D. M. Whittaker and I. S. Culshaw. Scattering-matrix treatment of patterned multilayer photonic structures. Physical Review B, 60 (4), 2610-2618, 1999.
- [30] R. C. Rumpf. Improved formulation of scattering matrices for semi-analytical methods that is consistent with convention. Progress In Electromagnetics Research B, 35, 241-261, 2011.
- [31] K. Busch, G. von Freymann, S. Linden, S. F. Mingaleev, L. Tkeshelashvili, and M. Wegener. Periodic nanostructures for photonics. Physics Reports, 444 (2-6), 101-202, 2007.